

QUANTUM GRAVITY

T. Goldman^{*}, J. Pérez–Mercader^{†*}, Fred Cooper^{*}
and Michael Martin Nieto^{*}

^{*}Theoretical Division, Los Alamos National Laboratory,
Los Alamos, New Mexico 87545

[†]Laboratorio de Astrofísica Espacial y Física Fundamental,
Apartado 50727
28080 Madrid

Submitted to **Nature**

ABSTRACT

We show how, by considering the cumulative effect of tiny quantum gravitational fluctuations over very large distances, it may be possible to: (*a*) reconcile nucleosynthesis bounds on the density parameter of the Universe with the predictions of inflationary cosmology, and (*b*) reproduce the inferred variation of the density parameter with distance. Our calculation can be interpreted as a computation of the contribution of quantum gravitational degrees of freedom to the (local) energy density of the Universe.

Study of the observed rotation curves of galaxies and their non-Keplerian fall-off has led to the conclusion that large amounts of non-luminous matter (Dark Matter) must exist, with a relative concentration that must increase as one moves away from the center of the particular galaxy. Furthermore, dynamical analysis of binary galaxies, groups of galaxies, clusters of galaxies and the structures known at the largest scales, show a slow, but clear, inferred increase in the ratio of mass to luminosity, M/L . This translates into a corresponding trend for the contribution of each structure to the inferred cosmological density parameter $\Omega = 8\pi G\rho/(3H^2)$, where G is Newton's constant, ρ is the average mass density in the Universe, and H is the Hubble constant.

Thus, larger amounts of dark matter are required as the scale of the structure grows. (See Figure 1 and Refs. 1–4.)

In addition, inflationary scenarios of the Early Universe, which solve (among others) the horizon and flatness problems, while maintaining the successes of the Big-Bang, predict that $\Omega = 1$. On the other hand, primordial nucleosynthesis of the light elements, based on the standard model of cosmology and known nuclear and particle physics, bounds the total baryonic density of the Universe to the range $0.010 \leq \Omega_B h^2 \leq 0.035$. Here h is Hubble's constant, in units of $100 \text{ Km s}^{-1} \text{ Mpc}^{-1}$, which is somewhere between 0.4 and 0.8. Thus, one can at most account for less than 10% of the energy density required to understand the dynamics of large scales. This is the Dark Matter (DM) Problem.

The above conclusions are based on classical general relativity and, where applicable, its non-relativistic limit, Newtonian gravity. In this letter we will study the impact that quantum-mechanical corrections to classical general relativity have on the DM problem. We will find that quantum gravity effects can account for a major portion of the effects attributed to DM and can also predict the value of Ω as a function of distance scale over a *wide* range of distances differing by more than 25 orders of magnitude, without having to introduce what is conventionally known as “Dark Matter”.

In quantum field theory, quantum effects lead to modifications of the parameters

of the theory due to the *unavoidable* presence of virtual processes. These modify the observed parameters and convert them into scale-dependent quantities. In general, the scale dependence of the corrections is logarithmic and their sizes are small, but if the range of distances involved is very large, then their effects can build up and become sizeable. For example, in Quantum Electrodynamics, QED vacuum polarization effects increase the value of α_{em} (the fine structure “constant”) from $\sim 1/137$ at atomic distances to^[5] $\sim 1/128.5$ at scales of the order of the Compton wavelength of the Z^0 -particle (2×10^{-16} cm). Similar corrections for QCD (the theory of the strong interactions) make α_{strong} decrease from ~ 0.179 , measured at the mass scale of the bottom quark (Compton wavelength 4×10^{-15} cm) to ~ 0.101 at the scale of the Z^0 . As is well known, these effects are predicted by the solutions to the renormalization group equations (RGE) for α_{em} and α_{strong} , and find spectacular confirmation in high energy particle physics experiments at SLAC, Fermilab and CERN.

These effects may be understood by a classical analogy using fluid dynamics. It is well known that for the same applied force, a longer ship achieves a higher limiting speed than a shorter ship of the same (mass and) cross-section presented to the water. Thus the force per unit area exerted by the fluid depends on the length scale of the “probe” used to measure it. Physically this occurs because the longer vessel reduces excitation of short wavelength modes of the fluid that would “close” behind the shorter ship. In the quantum case, coupling is to virtual modes for which the system lacks sufficient energy to produce a real excitation. The strength of this coupling still depends on the congruity between the scales of the modes and the probe, thus producing a scale dependence to almost all measured quantities.

We have asked the equivalent scale-dependence question for gravity and investigated how such effects would manifest themselves. In order to do that we start with the higher derivative theory of gravity described by the following action:

$$S = \int d^4x \sqrt{g} \left[\Lambda - \frac{R}{16\pi G} + aW + \frac{1}{3}bR^2 + \alpha_V R^* R^* + \kappa D^2 R \right] + S_{surface} + S_{matter}, \quad (1)$$

which has been studied in detail by, among others, Fradkin and Tseytlin^[6].

In this action, the R^2 -terms may be thought of as those necessary, at the classical level, to make the Einstein action (first two terms in Eqn.(1)) quantum-mechanically better-defined in regimes where the curvature becomes stronger. One obtains this action by expanding certain more complete actions in a functional Taylor series in derivatives of the metric. Furthermore, we expect that the physics described by this (or any reasonable) action to be independent of the scale at which one is performing the experiments and defining the physical quantities. Because of the R^2 -terms, the action is renormalizable and one can apply these arguments^[7].

In fact, one can construct a “Wilsonian” version^[8] of the action by substituting for the parameters appearing in the action their scale-dependent (so-called “running” or effective) equivalents, i.e., by substituting the solutions to the appropriate (one-loop in our case) renormalization group equations computed from this action. This Wilsonian action gives rise to “classical” equations of motion which contain the effects of the quantum corrections to the physics at each given scale. These corrections are incorporated in the values that the effective parameters (including Newton’s constant) take at each distance scale. Thus, if we are interested in measuring Newton’s constant at some distance, r , we must include the effect that (quantum) vacuum fluctuations have on the (micro) local curvature of space-time. These fluctuations produce ripples in space-time noticeable at distances of $O(10^{-33})$ cm. The ripples will be conspicuous at these distances, at which they can be “seen” as changes in the curvature of space-time or (equivalently) as a modification in the value of Newton’s constant. As we go past these very short distances, the fluctuations occurring at the shortest distances will still be felt and can be taken into account by using a value of Newton’s constant different from the one we used at the smaller scale. This is due to the fact that, unlike the case of electric charge, there is no screening mechanism for gravity, and that gravitation couples coherently to matter. Depending on how the (unavoidable) quantum fluctuations of the geometry affect the suitably averaged local curvature, the value at larger distances of Newton’s constant will grow with scale, decrease, or even oscillate! Repeated application of this procedure (very similar to the block renormalization procedure in statistical mechanics^[9]) leads to an effective Newton’s

constant which is a function of r , but which contains the effects due to those quantum gravitational fluctuations for which the correlation length is not larger than the length scale we are probing.

This variation with distance of G , or of any of the parameters entering in the lagrangian, is controlled by the singular behavior¹ of the appropriate correlation function which serves to define that parameter. For G , this may be obtained by computing the logarithmic divergences that quantum fluctuations induce on the R -term of the action. Carrying out this procedure for each of the parameters in the action of Eqn. (1), one arrives at a set of renormalization group equations valid for mass scales of less than 10^{-5} eV c⁻² corresponding to lengths longer than 1 cm. Here we have ignored photon and, possibly, neutrino contributions. The mass (and quantum mechanically corresponding length) scale is conservatively chosen to avoid including (the effects of) various additional degrees of freedom, which are expected to be very massive and which are required for controlling the renormalization behavior of the full theory at much shorter distance scales.

The effective parameters are given by the following solutions^[10]:

$$a(t) = a_0 + \frac{133}{10}t, \quad (2a)$$

$$\omega(t) = \frac{\omega_+ - \alpha\omega_- \left(\frac{a}{a_0}\right)^{-\gamma}}{1 - \alpha \left(\frac{a}{a_0}\right)^{-\gamma}}, \quad (2b)$$

$$\frac{G(t)}{G_0} = \left[\frac{\omega_0}{\omega(t)} \right]^{\alpha_0} \left[\frac{\omega(t) - \omega_+}{\omega_0 - \omega_+} \right]^{\alpha_+ + \alpha_-} \left[\frac{a(t)}{a_0} \right]^{\delta}. \quad (2c)$$

Here $t \equiv (32\pi^2)^{-1} \log(\frac{\mu^2}{\mu_0^2}) = (32\pi^2)^{-1} \log(r_0^2/r^2)$; $\omega_{\pm} = \frac{-549 \pm 7\sqrt{6049}}{200}$ are the two fixed points of $\omega(t)$; $\alpha \equiv (\omega_0 - \omega_+)/(\omega_0 - \omega_-)$; $\gamma \equiv \frac{100}{399}(\omega_+ - \omega_-) \sim 1.36448$; $\alpha_0 = -13/5$; $\alpha_+ + \alpha_- = -18/5$ and $\delta = -1.24253$. The quantity μ_0 (or r_0) is a reference *momentum*

¹To see the origin of this statement, consult, e.g., *Physical Kinetics*, Vol. 10 of the Landau and Lifschitz's Course of Theoretical Physics, Pergamon Press, New York.

(or *distance*) scale at which the parameter is matched to the experiment: $a(r_0) = a_0$, $\omega(r_0) = \omega_0$ and $G(r_0) = G_0$. The behavior of the effective couplings $a(t)$ and $\omega(t)$ for $a_0 = 5.67$ and for different initial values ω_0 are shown in Figure 2.

The parameters a and b ($\equiv -\omega a$) are constrained by laboratory and geophysical experiments^[11] as well as by precise measurements² of the orbital precision of Mercury. For the gravitational potential due to a mass m , obtained from the static piece of the graviton propagator, one finds in configuration space

$$V(r) = \frac{-G(r)m}{r} \left[1 - \frac{4}{3}e^{-m_2(r)r} + \frac{1}{3}e^{-m_0(r)r} \right], \quad (3)$$

where $m_2^2(r) = (16\pi G(r)a(r))^{-1}$ and $m_0^2(r) = (16\pi G(r)\omega(r)a(r))^{-1}$. From these expressions one obtains the following formula for Newton's constant measurable in a Cavendish type experiment:

$$G_N(r) = G(r) \left\{ \left(1 - \frac{\partial \ln G(r)}{\partial \ln r} \right) \left[1 + \sum_{k=0}^1 \gamma_{2k} e^{-r/\lambda_{2k}} (1 + r/\lambda_{2k}) \right] - \sum_{k=0}^1 \gamma_{2k} e^{-r/\lambda_{2k}} \frac{r}{\lambda_{2k}} \frac{\partial \ln f_{2k}}{\partial \ln r} \right\}, \quad (4)$$

where $f_0 = \omega(r)a(r)$; $f_2 = a(r)$; $\lambda_0 = \hbar/(m_0(r)c)$; $\lambda_2 = \hbar/(m_2(r)c)$; $\alpha_0 = +1/3$ and $\alpha_2 = -4/3$. Clearly, for values of $\omega(r)$ and $a(r) \leq O(1)$, the contribution from the Yukawa (exponential) terms is negligible at distances of a few times Planck's length ($\sim 10^{-33}\text{cm}$). Thus, we can safely ignore their contribution to the effective value of Newton's constant.

Another point to bear in mind is that the gravitational degrees of freedom contained in this action beyond the spin-2 graviton, and which guarantee renormalizability, could at first sight lead to problematic behaviors^[14]. The presence of a spin-2 massive ghost (affectionately known as a "poltergeist") with mass m_2 can lead to loss

²Notice that the usual PPN approximation^[12] does not apply here^[13], since the effective potential, Eqn. (3) below, has finite range terms.

of unitarity³ in both the high and low energy regions. In addition, the massive scalar could mean the presence of an instability in the vacuum. The resolution of these issues lies beyond the classical theory, in that, to correctly understand their import, one has to consider the effects of quantum corrections. These can come to the rescue since they significantly affect the energy dependence of the mass which now has to be promoted into a scale-dependent effective mass.

From the analytic expression for the mass of the poltergeist, one sees that the attractive characteristic of gravity ($G > 0$) makes it into a bona fide, real mass, for $a(r) > 0$. Furthermore, in the infrared a goes to zero, $G(r)$ increases only mildly, and $m_2^2 \rightarrow \infty$, so the effects of the poltergeist are *strongly* suppressed. In the ultraviolet, where the problems from unitarity become acute, quantum corrections also come to the rescue. For gravity in its asymptotically free regime (i.e., for parameters such that $G(r) \rightarrow 0$ as $r \rightarrow 0$), m_2^2 grows *faster* than $1/r^2$ and the effects of the poltergeist are subdominant at high momentum transfer.

The remaining extra mode, the scalar field with mass m_0 , will be tachyonic if m_0^2 is negative. In ordinary quantum field theories this implies that, at the length scales on which the scalar field is active, the vacuum around which the quantum field theory is built does not correspond to the lowest energy state for the system. Here the same arguments we used for the poltergeist can be applied, decoupling the tachyon in both the infrared and the ultraviolet and thus rendering the tachyon harmless in both regions. In addition to needing asymptotic freedom in G for a correct behavior in the ultraviolet, one also needs to guarantee that $\omega(r)$ remains finite. This is readily seen to be true from the renormalization group equation for $\omega(r)$. In the intermediate scale region the coupling to matter fields can change the sign of m_0^2 and bring the system to its true vacuum state. After the decoupling^[16] of the matter fields takes place, the infrared regime sets in and the decrease of $a(r)$ at large r decouples the scalar from the effective low energy physics.

From the above discussion, we conclude that we must stay in the regions of pa-

³It is worth mentioning that some quantum gravitational processes are known to violate *naive* unitarity. One such process is the Hawking evaporation of a black hole^[15]. The correct implementation of unitarity in quantum gravity remains an open question.

parameter space where $a > 0$ to have unitarity, whereas ω can assume both positive and negative values.

Applying Wilsonian arguments to the basic quantum gravitational action of Eqn. (1), under the ansatz of a homogeneous and isotropic metric, one finds that, for comoving distances greater than 1 cm, the energy density parameter for the associated matter-dominated Universe, Ω , is promoted into a scale-dependent quantity $\Omega(r)$, given by:

$$\Omega(r) = \frac{8\pi}{3H_0^2} G_N(r) \rho_m. \quad (5)$$

(Because of the decoupling theorem^[16], charge screening and the distance scales we are considering, ρ_m is unaffected by quantum corrections at the one loop level).

In this expression $G_N(r)$ is given to a very high accuracy by

$$G_N(r) = G(r) \left[1 - \frac{\partial \ln G(r)}{\partial \ln r} \right]. \quad (6)$$

We can write Eqn. (5) in the following reparametrization

$$\Omega = \frac{8\pi}{3H_0^2} G_{lab} \delta(r, r_0) \rho_m, \quad (7)$$

where $\delta(r, r_0) = G_N(r)/G_N(r_0)$ and r_0 is the reference laboratory distance at which G_{lab} is measured.

General quantum field theoretical reasoning applied to gravity and to the basic equations of cosmology thus leads one to the conclusion that, to compare the inferred values of the density parameter for different (size) structures in the Universe with the corresponding values predicted by theory, one must use the “improved” expression given in Eqn. (7). By construction, this takes into account the unavoidable, scale-dependent effects of “microscopic” quantum fluctuations in the geometry. The existence of a region in parameter space (a_0, ω_0) where the function $\delta(r, r_0)$ is a growing function of r , together with Eqn. (7), opens up the possibility of understanding the observational data on Ω over a wide range of scales and reconciles the inflationary

Universe scenarios with observation. In fact, within a very ample region of parameter space, one can reach this agreement without having to introduce any more contributions to ρ_m than what is consistent with the primordial nucleosynthesis of the light elements. This means that we *need not* add any extra, exotic contributions to the fraction of the critical energy density, and thus do not need what is commonly known as dark matter to understand the successes of the inflationary scenario.

The fraction of the critical energy density is constrained by nucleosynthesis data to be^[18] $0.017 \leq \Omega_B \leq 0.1$ or^[4] $0.02 \leq \Omega_B \leq 0.22$. We saturate the nucleosynthesis upper bound (which we set at $\Omega_B \leq 0.16$) and produce the fit shown by the continuous curve of Fig. 3, which is a plot of the function $\delta(r, r_0)$, for $r_0 = 1$ cm. At once we see that for most of the typical systems, the inferred values of $\Omega(r = \text{Typical Size})$ fall below our predicted values. This means that our scheme allows less mass to be contained in these structures than what is commonly inferred for them by dynamical, geometrical and (certainly) luminosity methods.⁴

Finally, it should be emphasized that our calculations take as a starting point the model of quantum gravity of Eqn. (1) which, although minimal, is nothing but one model in a class of models. It is conceivable that other models⁵ yield different behaviors for $\Omega(r)$. In addition, we have not included the effects on the cosmological constant which, following the usual practice, we set equal to zero. However, preliminary results in this problem^[17], along the lines of what has been described in this paper, show that for the model of Eqn. (1) the same mechanism that makes G grow with distance has the potential to produce an exponentially strong suppression of the

⁴Other independent tests are the ones based on the age of the Universe combined with Geometry. These (i) assume the validity of the Standard Model and (ii) must necessarily rely on H , whose value depends on a variety of astrophysical assumptions. Such tests will be more meaningful once purely geometrical measurements are available from, for example, Hipparcos, the parallax measuring ESA satellite. We also see that quantum gravitational effects incorporated via the effective couplings provide a natural framework for *understanding* the different values of $\Omega(r)$ observationally inferred for the various typical structures.

⁵Notice that in the present model, and for the chosen parameter values, G changes by less than a factor of 5 over 28 decades of distance. This perturbative leading logarithm change should be compared with the equivalent change in grand unified theories, where the effective couplings change (e.g. the strong coupling constant) by a little more than a factor of 7 in 14 decades in distance^[5]. This more gentle growth for the gravitational coupling is consistent with the intuitive notion of gravity as a much weaker force than the others.

physical value of the cosmological constant from its value at scales corresponding to much less than 10^{-5} eV c⁻² to the largest known distances, of the order of 10^6 kpc. This adds weight to our contention that many issues of large scale astrophysics, not just Dark Matter, require the inclusion of quantum mechanical effects on gravitation before they can be properly evaluated and resolved.

References

References

- [1] Ostriker, J. P. , Peebles, P. J. E. and Yahil, A. , in *Ap. J.* **193** (1974) L1–L4.
- [2] Peebles, P. J. E. , in *Physical Cosmology*, eds. Balian, R. , Audouze, J. , Schramm, D. (North–Holland, Amsterdam, 1980), pp. 213–267.
- [3] Binney, J. and Tremaine, S. , *Galactic Dynamics* (Princeton University Press, Princeton, 1987).
- [4] White, S. D. M. , in: *Physics of the Early Universe, SUSSP 1989*, eds. Peacock, J. A. , Heavens, A. F. and Davies, A. T. (Edinburgh University Press, Edinburgh, 1990), pp. 1–62.
- [5] Goldman, T. and Ross, D. A. , *Phys. Lett.* **84B** (1979) 208–210; *Nucl. Phys.* **B162** (1980) 102–114.
- [6] Fradkin, E. S. and Tseytlin, A. A. , *Nucl. Phys.* **B201** (1982) 469–491.
- [7] Stelle, K. S. , *Phys. Rev. D* **16** (1977) 953–969; *Gen. Relativ. Gravit.* **9** (1978) 353–371.
- [8] Wilson, K. G. , *Phys. Rev.* **179** (1969) 1499–1512.
- [9] See, e. g., Parisi, G. , *Statistical Field Theory* (Addison–Wesley, New York, 1988).
- [10] Goldman, T. , Pérez–Mercader, J. , Cooper, F. and Nieto, M. M. , *Phys. Lett.* **B 281** (1992) 219–224; in: *The Vancouver Meeting – Particles and Fields '91*, vol. 2, eds. Axen, D. , Bryman, D. and Comyn, M. (World Scientific, Singapore, 1992).

- [11] Nieto, M. M. and Goldman, T. , *Phys. Rep.* **205** (1991) 221–281; Fishbach, E. and Talmadge, T. , *Nature* **356** (1992) 207–215.
- [12] Will, C. , *Phys. Rep.* **113** (1984) 345–422.
- [13] Helbig, T., DESY–Preprint *DESY 91–002*, January, 1991, 19pp.
- [14] For a lucid and pedagogical presentation of some of the issues related to ghosts cf. Hawking, S. , in: *Quantum Field Theory and Statistical Mechanics*, vol. 2, eds. Batalin, I. , Isham, C. and Vilkovisky, G. (Adam and Hilger, Bristol, 1987), pp. 129–139.
- [15] For a discussion, see Strominger, A. , in: *Quantum Theory of Gravity*, ed. Christensen, S. (Adam and Hilger, Bristol, 1984), pp. 210–223.
- [16] Appelquist, T. W. and Carazzone, J. , *Phys. Rev. D* **11** (1975) 2856–2861.
- [17] Barbero, F. , Cooper, F. , Goldman, T. , Nieto, M. M. and Pérez–Mercader, J. , *in preparation*.
- [18] Walker, T. , *et al.*, *Ap. J.* **376** (1991) 51–69.

Figure Captions

Fig. 1 Plot of the inferred density parameter Ω_0 for different typical structures. This plot is an update of the one given in Reference [1]. The data plotted are extracted from that given by References [2], [3] and [4]. The different structures are: *(A)*, Solar Stellar Neighborhood; *(B)*, Visible Galaxies; *(C)*, Extended Rotation Curves; *(D)*, Binary Galaxies; *(E)*, Galaxy Groups; *(F)*, Rich Clusters; *(G)*, Virgo Infall; *(H)*, IRAS Galaxies.

Fig. 2 The scale evolution of the effective couplings $a(r)$ and $\omega(r)$ in the renormalizable theory of gravity considered in the text and described by Equation (1). The running of the coupling ω is shown for different initial values. Lines showing the two fixed points, ω_+ and ω_- , are also drawn.

Fig. 3 The product of the ratio of the nucleosynthesis upper bound on the (baryonic) matter density to the critical matter density with the ratio of the scale dependent value of Newton's constant to the laboratory value, as a function of scale, for a point mass, compared with data on the value of the deceleration parameter inferred using the laboratory value of Newton's constant. From $10^{-7}kpc$ down to laboratory scales ($3 \times 10^{-22}kpc$), the curve is visually indistinguishable from a horizontal straight line. A horizontal line marks the nucleosynthesis upper bound on the contribution of baryonic matter to the critical density.